Sovereign Borrowing for Dubious Reforms
A model with applications on the EMU

Maximilian Ludwig
Department of Economics, Hamburg University
Von-Melle-Park 5
20146 Hamburg, Germany
mludwig@mludwig.org
Sovereign Borrowing for Dubious Reforms
A model with applications on the EMU

Maximilian Ludwig∗
April 22, 2013

Abstract
This paper presents a model where governments need loans to finance reforms and may misuse these funds for consumption without immediately exposing this to its lenders. Such a misuse is ultimately followed by a sovereign default, therefore lenders will try to discipline governments in favor of true reforms. This puts a government and its lenders in a sequential game, which has two remarkable properties: First, discipling a government in favor of reforms may work, albeit far from perfect. Second, the game implies jumps in the interest rate as observed in the EMU debt crisis.

JEL Codes: C73, F34, H63

Keywords: Reforms, Sovereign default, Sequential Game, Bail-Out

1 Motivation and Synopsis
Amongst the many puzzling outcomes of the recent financial crises, the pattern of differences and similarities between EMU countries that underwent a near default is particular confusing. Both the level of debt and the financial track record of the affected governments are so grossly different that it is hard to give a simple explanation on why these particular governments went into trouble. An important property shared by all southern European countries affected by the crisis is however a large backlog of reforms. This is particularly explicit for Greece and Portugal. In case of Greece, an EU report on the implementation of EU policy guidelines (European Comission, 2005, pp. 131) found that about all recent policy measures taken in the fields of improving public finances, reducing structural unemployment by labor market reforms as well as increasing productivity

∗University of Hamburg, Department of Economics, mludwig@mludwig.org. The latest version of this paper, as well as MATLAB code for numerical examples, is available at mludwig.org/research.html.
had limited success. Depressingly, the same list of policy measures reappears in the economic adjustment program requested by the EU for its assistance (European Commission, 2010b), suggesting a complete failure in the following years.

To a very large extend, the same is true for Portugal. Amongst other things, Portugal has been able to keep the general government deficit below 3% only because of one time revenues and had very limited success in boosting productivity to counteract high wage growth (European Commission, 2005, pp. 205). Therefore both an improvement of government finances and labor productivity reappear in list of policy measures demanded from Portugal by the EU (European Commission, 2011).

Occurrences of debt crisis in an environment of failed reforms can also be found for countries outside the EMU, e.g. for the default of the Ukraine. After a slow start both academic observers and the IMF saw progress in the reform measures implemented by the Ukrainian government in the mid 1990s (cf. IMF, 1995, Ishaq, 1996). The implementation of monopoly regulation, the privatization of state owned firms and the liberalization of trade were on top of the reform agenda (cf. Ishaq, 1996, pp. 27). However, in the aftermath of the Russian crisis in late summer 1998, the Ukraine began to negotiate haircuts for its T-Bills and called the IMF for a stand-by agreement (cf. Sturzenegger and Zettelmeyer, 2007, pp. 121). Key policy measures announced to the IMF under the extended arrangement (cf. The Government of the Ukraine, 1998) are again the regulation of monopolies, privatization and the liberalization of prices, suggesting that the implementation of these measures in the mid 1990s had very limited success.

These observations raise the suspicion that the combined occurrence of failed reforms and debt crisis might not be a mere coincidence but rather the result of a hidden causal relation. The literature concerning this link is sparse. There is a well developed strand of empirical literature that studies reforms in the EMU both theoretically and empirically but with the focus on links between labor market reforms and output / inflation and unemployment.1 A smaller and quite diverse strand of applied and empirical works, mainly located in law, development economics and political science, studies the redesign of tax systems with the focus on reducing structural deficits and / or increasing government’s funds. In terms of theoretical models, there exists only one contribution that investigates the link between reforms and government finances: The work of Andres Velasco (1997). In his model, a budget deficit emerges in the interaction of interest groups with different objectives and forces them to conduct stabilization policies beneficial in the long run. The possibility of a default of the government is however excluded from the model.

This paper addresses a different question: can lenders discipline governments in favor of reforms if they provide the resources that enable governments to conduct them? Or, put differently, may there be a kind of market discipline which forces governments to conduct reforms? They answer is yes, albeit this discipling works far from perfect.

---

1A great chunk of this literature is reviewed by Beetsma and Giuliodori (2010).
A core feature of the model I shall develop is that the interest rate does not behave smoothly but jumps between two regimes: one where no risk premium is charged from the government and one where a very high premium is charged. This property helps to provide an alternative explanation for interest rate jumps as observed in the EMU debt crisis.

The model is build around the possibility of a sovereign default. This provides an incentive for lenders to care about the use of the resources provided by them. Trivially, lenders with a perfect ability to observe governments’ reform action can easily design strategies that force any government into true reforms. In the real world, such an ability does not exist. An obvious example is Greece, where irregularities in and a low quality of economic data are well documented (European Comission, 2010a). But this rather bold distortion may distract one from the fact that the very nature of reforms makes it difficult to evaluate whether a particular measure is appropriate to meet its promised ends. Does for example a new organizational form of regional governance really require new office buildings and an expensive new computer system? Does the local firm that provides the computer system charge a reasonable price? These questions may already be hard to answer for employees of the central government, a foreign investor who considers to buy a municipal bond that finances these measures will struggle even more.

An important issue of reforms is timing. Timing may introduce ambiguity whether a government truly reforms or does not. This is because a government might delay its reforms by initially spending little, giving the pledge to raise spending afterwards. For simplification such ambiguity is ruled out by demanding a certain minimum reform spending. Governments that fail once to provide this spending are assumed to fail in their reforms. This implies the government can either be truly reforming or not, there is no ”maybe”.

Lenders who can provide loans to governments for reforms will try to identify whether the government truly reforms. This puts lenders and the government in a sequential game. A classic application for such a game comes from microeconomics: in the Kreps and Wilson (1982) monopoly entry game, entrants try to infer from a monopolist’s action whether the monopolist is weak (unable to endure price wars) or strong (able to endure price wars). The insight that the interaction between a government of unknown type (not willing to default / willing to default) and its lenders can have a structure similar to the Kreps-Wilson model has been exploited by Harold Cole and Patrick J. Kehoe (henceforth CK) in a series of papers (Cole and Kehoe, 1995, 1997, 1998).

The model developed in this paper builds on CK’s work: a sequential game very similar to CK’s model is at the core of my model, but here neither government’s type nor the initial prior used by lenders when learning about the government is exogenous. To signal this difference, I talk about government’s class instead of type and of lenders’ belief instead of their prior. The broad time structure of the model is as follows:

---

In the next section, I define the economic environment. This is followed by an exposition of the sequential game. After this exposition, I study the determinants of government’s choice and a rational strategy for lenders to build an initial belief. After a brief conclusion on this basic model, I study the effect of the possibility of a bail out by another government and use the findings to provide an alternative interpretation for the history of intra EMU interest rate spreads.

2 Model

2.1 Setup

Assumptions 1 to 7 characterize the government and its resources:

**Assumption 1 (Preferences).** The government seeks to maximize its expected utility given by

\[ U_0 = \sum_{t=0}^{\infty} \beta^t E_0[c_t] \]

where \( c_t \) is government consumption and \( 0 < \beta < 1 \) the subjective discount factor.

**Assumption 2 (Technology).** The income of government \( j \) is generated by a linear technology with deterministically fluctuating productivity.

\[ y_t^{(j)} = \phi_j A_t b_{t-1} + m_t \]

where \( A_t = \begin{cases} A > 0 & \text{if } t \text{ is even and } \leq T \\ 0 & \text{otherwise} \end{cases} \)

\( b_{t-1} \) denotes government borrowing in the previous period and \( m_t \) the output of the so called mock up technology I discuss below. \( \phi_j \geq 1 \) is an idiosyncratic parameter that describes the ability of government \( j \) to convert a loan into income. I call this parameter the efficiency of government \( j \). At the beginning of time, this parameter is drawn from an exponential distribution whose parameter is publicly known. Viz, all agents know that \( P(\phi_j \leq k) = 1 - \exp(-\lambda(k-1)) \). \( \lambda \) controls the probability mass of high efficiencies.

The pattern of \( A_t \) is a very simple way to generate a constant need for loans. To generate income, the government will borrow in uneven periods. It can not do so in even periods because in the next period \( A_t = 0 \), viz the government has zero income independently of the loan size and can thus neither consume nor repay its debt.
Assumption 3. In period 0, which is defined to be an even period, government income is given by $\phi_j A_t + m_t$.

Assumption 4 (Reforms). Reforms require the government to spend $s > 0$ on reforms in every even period, from period 0 to period $T$. If the government does so and honors all contracts, it gets access to a perpetual income of $\psi \phi_j A > 0$ from period $T + 1$ on.

$\phi_j$ also determines the future benefits of reforms because idiosyncratic characteristics of an economy are, if at all, changing very slowly. Therefore the benefits from reforms are determined by the very same characteristics that determine the difficulty of undertaking reforms in the first place.

Due to assumption 4, reforms have no observable effect up to period $T + 1$. This allows a government to just declare that it spends $s$ on reforms and use a large part of $s$ to increase consumption. But because a government will only repay the last loan if it benefits from it due to successful reforms, lenders will not lend to a government that does not reform. This is simply because if the government defaults with probability one in some future period, some lenders will have to endure a loss. Therefore they will not offer a loan to the government which makes the government default on the previous loan and so on. Thus, a government must hide its unwillingness to reform to get access to loans. As exemplified by Greece, governments are indeed able to hide their true actions, at least for a certain time period. This is introduced into the model by the mock-up technology:

Assumption 5 (Mock-Up Technology). A government that does not prefer to reform needs to generate the image to lenders that it is truly reforming. I denote spendings on true reforms as $(1 - \alpha_t)s$. The rest of government’s reform spendings, namely $\alpha_t s$, is allocated in the mock-up technology.

The output of the mock-up technology is given by $m_t = \gamma \alpha_t s$ where $0 < \gamma < 1$ scales its productivity. For simplicity, I assume that any $\alpha_t > 0$ is sufficient for the government to appear to lenders as indeed spending $s$ on true reforms.

The intuition for the mock-up technology is as follows: A government may undertake costly reforms of its institutions and improve the economy’s infrastructure which, if accomplished, implies substantial long term benefits. But the government can also mimic reforms by routing the spendings to its employees and opaque government entities and organize cover up measures, e.g. by manipulating national accounting figures. In the latter case, government consumption is larger than the consumption possible for a truly reforming government with the same efficiency. At the same time, the government appears as spending $s$ to lenders, which is only possible for a government with a larger income, viz larger $\phi_j$.

Assumption 6 (Assets). A government can borrow from lenders defined below but can not lend to them. A government can not commit to the repayment of a loan. Govern-
ment’s decision to repay is denoted as \( z_t \in \{0, 1\} \), where \( z_t = 1 \) means that the government repays and \( z_t = 0 \) means default. If in period \( t \) no repayment is due, \( z_t = z_{t-1} \). The aggregate loan size is denoted as \( b_t \) and the gross interest rate as \( R_t \). \( b_t \) must not exceed an upper bound which is normalized to one.

**Assumption 7.** Any government is able to repay a loan with an interest rate slightly higher than the risk free interest rate \( \rho \equiv 1/\beta \) and spend \( s \). That is, I assume

\[
A = \rho + s(1 + \nu)
\]

where \( \nu > 0 \) scales the ability of the least efficient government to bear risk premia.

The last assumption, combined with risk neutrality, implies that the government will always prefer to use the full loan to generate income instead of consuming a part of the loan right away. To write down a budget constraint for uneven periods, define \( x_t \) as "investment-use" of the loan \( b_t \) to generate income in the next period. The budget constraint is then \( c_t + x_t = b_t \) and the government will choose \( c_t = 0 \).

In even periods the government receives income, might repay the loan and spend on reforms and consumes what is left. Obviously, a government that defaults has no advantage from spending on reforms, therefore \( z_t \) describes both government’s default and reform spending decision. This allows to denote the budget constraint for even periods as follows:

\[
\phi_j A_{t-1} b_{t-1} = z_t (b_{t-1} R_{t-1} + s - m_t) + c_t
\]

**Definition 1 ("types" /classes of governments).** To signal the difference to standard sequential games, I replace the term "type" with the term "class". Because ambiguity on the nature of governments’ willingness to reform is ruled out by assumption, only two classes of governments exist: Those who prefer to reform, denoted as class \( r \), and those who do not, denoted as class \( n \).

The following four assumptions characterize the lenders:

**Assumption 8 (Existence).** In every period, a large number of lenders enters the model. Each lender has a wealth \( W \geq 1 \) and leaves the model at the end of the next period. They are not affected by government’s actions beyond the repayment of their loan, possibly because they live in another country.

**Assumption 9 (Preferences).** Lenders maximize

\[
V_t = c_t^{(l)} + \beta E_i [c_{t+1}^{(l)}]
\]

subject to \( W \) and their investment opportunities. \( \beta \) is the same subjective discount factor as used for the government.
Assumption 10 (Investment Opportunities). Lenders have access to the international capital market where they can do risk free investments that yield \( \rho \equiv \frac{1}{\beta} \) and can lend to the government. The combination of loan size and interest rate offered by a lender to the government will be called a contract. Loans can be of size zero if lenders find it preferable to lend only to the international capital market. Let \( \Lambda_t \) denote the set of loan contracts offered by lenders in period \( t \).

Assumption 11 (Lender’s information set). Each generation of lenders has access to the time series published by the government, that is the history of government income, consumption and reform spendings. They also know all previous \( \Lambda_t \) as well as previous lenders’ belief about the class of the government, summarized in the probability \( p_t \equiv P(\text{gov’t is class } r \mid \Omega) \), where \( \Omega_t \) is lenders’ information set. That is, I assume that lenders entering the game in period \( t \) know

\[
\Omega_t \equiv \{y_{t-1}, c_{t-1}, z_{t-1}, b_{t-1}, R_{t-1}, p_{t-1}, \Lambda_{t-1}, \Omega_{t-1}\} \quad \text{where } t \geq 2
\]

Before any action, nature is assumed to draw \( \phi_j \) and publish \( A_t, \gamma, \psi, \nu \) and \( s \). The information set used by the first generation of lenders who provide the initial loan to the government is therefore defined as \( \Omega_1 \equiv \{y_0, c_0, \theta\} \) where \( \theta \) contains the publicly known parameters and knowledge about the environment just described.

The sequence of events is as follows:

\( t = -1 \): Nature draws \( \phi_j \), publishes the structure and parametrization of the environment.

\( t = 0 \): Government receives its initial income and makes sure lenders observe \( s \).

\( t = 1 \): Lenders look up \( \Omega_1 \), form initial belief on the class of the government and offer an initial set of contracts. The government accepts a subset of these contracts.

\( t = 2 \):

1. A generation of lenders enters the model that does not lend to the government.
2. Government receives output, decides on \( z_2 \) and consumes.
3. Lenders from period 1 may receive principal and interest and leave the model, the current generation updates \( p_1 \) to \( p_2 \).

\( t = 3 \): The next generation of lenders looks up \( \Omega_3 \) and offers new loan contracts, the government accepts a subset of contracts.

\( \ldots \) 2 and 3 repeat until \( T \) is reached.

**Lemma 1.** Lenders charge the risk adjusted interest rate \( R_t = \frac{1}{\beta P(z_{t+1} = 1 \mid \Omega_t)} \)
Proof. Each lender has to find the optimal allocation of her wealth $W$ in asset classes and over time. The optimization problem is

$$\max_{c_t, \omega} V_t \equiv c_t^{(l)} + \beta c_{t+1}^{(l)}$$

s.t. $c_t^{(l)} = \omega R_t \times P(z_{t+1} = 1 \mid \Omega_t) \times (W - c_t) + (1 - \omega) \frac{1}{\beta} (W - c_t)$

The first order condition for the allocation of savings is:

$$\beta R_t \times P(z_{t+1} = 1 \mid \Omega_t)(W - c_t) = (W - c_t) \Leftrightarrow R_t = \frac{1}{\beta P(z_{t+1} = 1 \mid \Omega_t)}$$

The first order condition for the allocation over time is

$$0 = 1 - \beta \omega R_t P(z_{t+1} = 1 \mid \Omega_t) - (1 - \omega) \Leftrightarrow R_t = \frac{1}{\beta P(z_{t+1} = 1 \mid \Omega_t)}$$

To get access to loans, any government must make sure lenders have reasons to believe that it is class $r$. To understand how it must behave, inspect period 1, where the first loan is offered. Lenders look up $y_0$ and infer that $y_0$ is consistent with the efficiency $\hat{\phi} = \frac{\gamma_0}{A} = \phi_j + \frac{\gamma s}{A} \alpha_0$, which can be generated by various combinations of $\phi_j$ and $\alpha_0$. $\hat{\phi}$ will be referred to as the observed efficiency. A class $r$ government generates $\hat{\phi} = \phi_j$ and thus a constant observed efficiency. This forces a class $n$ government to keep $\hat{\phi}$ constant by adjusting $\alpha_t$, given it prefers access to further loans.

Lemma 2. A government that chose $\alpha_0 \in [0, 1]$ has to set $\alpha_t = \alpha_0 b_{t-1}$ to generate a constant $\hat{\phi}$.

Proof. Lenders know output in period 0. The government generates the observed efficiency $\hat{\phi} = \phi_j + \frac{\gamma s}{A} \alpha_0$. If lenders offered a loan of size $b_{t-1}$, a class $n$ government has to adjust $\alpha_t$ to make sure that the observed efficiency is constant:

$$\left(\phi_j + \frac{\gamma s \alpha_0}{A}\right) Ab_{t-1} = \phi_j Ab_{t-1} + \gamma s \alpha_t \Leftrightarrow \alpha_t = \alpha_0 b_{t-1}$$

This implies that lenders can not identify a government’s class by varying the loan size because a class $n$ government appears exactly identical to a class $r$ government in terms of observed efficiency.

Lemma 2 also states that $\alpha_0$ determines $\alpha_t$ for the rest of the game. This means that once the government has chosen $\alpha_0$, the only degree of freedom it has to set $z_t$.

Definition 2 (Strategies). Lemma 2 implies that the government must choose its class in period 0. A strategy for the government consists therefore of a choice function that determines its choice of class. If the government prefers to be class $r$, it must set $\alpha_t = 0$ and $z_t = 1$ for all $t \in [0, T]$. If the government prefers to be class $n$, it can choose
different values. The strategy therefore also contains sub-strategies to set $\alpha_t$ and $z_t$ given a specific choice of class.

After period 1, the strategy of a lender is the set of contracts she offers after looking up $\Omega_t$. In period 1, lenders also have to find a rational initial belief about government’s class, viz their strategy also consists of a policy $p_1(\Omega_1)$.

**Definition 3 (Bayesian Equilibrium).** In equilibrium

- Given $\theta$, $\phi_j$ and the strategies of lenders, the strategy of a government maximizes its expected utility subject to the budget constraint, $b_t \leq 1$ for all $t$ and lenders’ belief.
- Given $\Omega_t$, the strategies of other lenders and the strategy of the government, each lenders’ strategy maximizes her expected utility.
- Lenders’ belief is updated using Bayes’ rule: if lenders in $t$ see $z_t = 1$, viz a repayment of their loan plus interest, they update the belief according to
  \[ p_t = \frac{p_{t-1}}{p_{t-1} + \sigma_{t-1}^n(1 - p_{t-1})} \]
  where $\sigma_{t-1}^n$ is the mixing probability chosen by a class $n$ government in period $t - 1$.
- The first generation of lenders chooses $p_1(\Omega_1)$ to maximize their expected utility and conditional on strategies of succeeding generations of lenders and government’s strategy in rational expectation of $p_1(\Omega_1)$.

### 2.2 The Sequential Game

#### 2.2.1 Structure

Lenders know that a class $n$ class government will default in $T$ at the latest. This means that in period $T$, $P(z_T = 1 \mid \Omega_{T-1}) = P(\text{gov’t is class } r \mid \Omega_{T-1})$. They will offer a loan for the last period if the risk adjusted interest rate allows a class $r$ government to repay and spend $s$, viz if

\[ \hat{\phi}Ab_{T-1} - \frac{\rho}{p_{T-1}}b_{T-1} - s \geq 0 \iff p_{T-1} \geq \frac{\rho}{\hat{\phi}A - \gamma b_{T-1}} \]  

(2)

where $p_{T-1} \equiv P(\text{gov’t is class } r \mid \Omega_{T-1})$. Now consider the case that the first generation of lenders found $p_1 > \frac{\rho}{\hat{\phi}A - \gamma b_{T-1}}$ to be rational for the initial loan. If all subsequent generations of lenders offer loans of size 1 too, the belief of lenders does not have to be updated throughout the game. Bayes’ rule implies that if no updating is necessary, no mixing of class $n$ governments is required (more on this later). But if such a government repays for sure rather than with a nonzero probability, there is no default risk and lenders will
offer contracts with $R_t = \rho$ for $t \in [1, T - 3]$. I shall call these contracts "No-mixing contracts".

This is the simplest variant of the sequential game. A class $r$ government repays for sure, a class $n$ government too except for the last period, both are therefore charged no risk premium except for the last period. One question remains: why should lenders offer loan contracts that yield an aggregate loan size of 1, viz at the upper bound? The intuition is simple: consider lenders in period 3, $z_2 = 1$ and $p_2 > \frac{\rho}{\hat{\phi} A - s}$. Let a lender offer a contract with the smallest possible loan size $b_3 = \frac{z_2}{A - p}$. If $b_3 < 1$, another lender could offer $b'_3 > b_3$ together with $R'_3 \geq \rho$. This contract implies an economic profit for the lender and is strictly preferred by the the government for a large set of possible $R'_3$ and $b'_3$. Therefore, competition between lenders has only one equilibrium in terms of $b_3$: $b_3 = 1$. This process repeats in all periods until $T - 3$ and works for $R_t > \rho$ too.

Consider now the case that $p_1 < \frac{\rho}{\hat{\phi} A - s}$. Lenders are now so skeptical about the nature of government’s reforms that if their belief would not be updated until period $T - 1$, the risk premium would be so large that a class $r$ government can not repay it and spend $s$ in period $T$. This would destroy its prospect of a perpetual income from the next period on and make it default too. This means that, given such a small initial belief in the government being class $r$, there exists no equilibrium in pure strategies except $b_t = 0$ for all periods.

Yet, if a class $n$ government uses a mixed strategy and this is rationally expected by lenders, there exists another equilibrium that is much more beneficial for governments. The economic intuition for this is relatively simple: suppose that the government begs lenders to offer it a loan contract with a positive loan size and lenders get soft an offer one. To curb expected losses, they must charge a risk premium as large as possible but small enough to allow a class $r$ government to repay and spend $s$. This is the case if $R_{t-1} = \hat{\phi} A - s$. But this contract leaves the government with zero consumption and due to $p_1 < \frac{\rho}{\hat{\phi} A - s}$, there is no hope for it to be offered any contract with positive consumption during the rest of the game. In this setting, a class $n$ government will see no benefit from repaying since it does not gain a perpetual income due to reforms after period $T$ and will therefore default. A class $r$ government will not. This means that lenders can "test" the willingness a government to reform by offering it contracts with zero consumption. If they see that a government repays, it is likely to be class $r$.

However, rational lenders have no reason to "get soft" and offer a loan contract that implies an expected loss. Here, the mixed strategy of class $n$ governments comes in: if such a government does not default with probability 1 if only zero consumption periods lie ahead and this is expected by lenders, offering a positive loan to the government will not imply an expected loss and thus be done.

One may ask why a class $n$ government should ever forsake utility by not defaulting as soon as only zero consumption periods lie ahead. This is because except for a very low $p_1$ (a quantification of "very low" is given below) the game has a "phase of tranquility"
where any government will repay for sure and thus pay no risk premium. Such a phase
is however only possible if lenders do not expect a class \(n\) government to default for sure
at its end, which is ensured by the mixed strategy.

I shall now study the mechanism of the game in more detail. The condition for the
belief in the last period has already been noted as (2). Bayes’ rule implies:

\[
p_{T-1} = \frac{p_{T-2} \cdot \frac{p_{T-3}}{p_{T-2} + \sigma_{T-2}^2 (1 - p_{T-2})}}{\frac{p_{T-3}}{P(z_{T-2} = 1 | \Omega_{T-3})}} \iff P(z_{T-2} = 1 | \Omega_{T-3}) = \frac{p_{T-3}}{p_{T-1}}
\]

(3)

Repayment in \(T - 2\) is only possible iff

\[
\hat{\phi} A b_{T-3} - \frac{\rho}{P(z_{T-2} = 1 | \Omega_{T-3})} - s \geq 0 \iff P(z_{T-2} = 1 | \Omega_{T-3}) \geq \frac{\rho}{\hat{\phi} A - s / b_{T-3}}
\]

(4)

Combining (3) and (4) yields the inequality

\[
\frac{p_{T-3}}{p_{T-1}} \geq \frac{\rho}{\hat{\phi} A - s / b_{T-3}}
\]

(5)

replacing \(p_{T-1}\) with (2) yields then

\[
p_{T-3} \geq \left(\frac{\rho}{\hat{\phi} A - s / b_{T-3}}\right) \left(\frac{\rho}{\hat{\phi} A - s / b_{T-1}}\right)
\]

The steps that yielded (5) can be repeated for every period in which a loan is granted.
This implies that in period \(T - k - 1\), the belief must fulfill

\[
p_{T-k-1} \geq \prod_{t=T-k}^{T-1} \left(\frac{\rho}{\hat{\phi} A - s / b_{t}}\right)
\]

For the initial loan, \(k = T - 2\). I already argued that competition between lenders will
drive \(b_t\) to 1. The condition just derived reveals that if the first generation of lenders
expects \(b_t = 1\) in all future periods there a second effect: the necessary initial prior
required to offer a loan at all is decreased. To study the effect inspect \(p_1\) with \(b_t = 1\) for
\(t \in [1, 3, \ldots, T - 1]\):

\[
p_1 \geq \left(\frac{\rho}{\hat{\phi} A - s}\right)^{T/2}
\]

(6)

Now suppose that the initial generation of lenders found \(p'_1 = (\rho / \hat{\phi} A - s)^{T/2}\) to be appropriate. Obviously, this means that lenders do not have to update their belief right away but
they can wait until the next loan, viz offer a no mixing contract. So \(b_t = 1\) has two beneficial
effects for the government: it gets the largest possible loan and risk premia are charged
at the latest possible point in time, allowing for a "phase of tranquility".
For a compact notation, I shall denote the condition for the last period of the game as
\[ p_{T-1} \geq \frac{\rho}{\hat{\phi}A - s} \equiv \mathcal{P}(\hat{\phi}) \]  
(7)

This condition will be referred to as the terminal condition. To make the notation even more compact, I suppress the dependence of \( \mathcal{P} \) on \( \hat{\phi} \) until the discussion of lenders’ policy for the initial prior. The function which describes the mixing probability as function of time and \( \mathcal{P} \) is readily computed by replacing the beliefs in Bayes’ rule, yielding:
\[ \sigma_{T-T}^n = \frac{\mathcal{P} - \mathcal{P}^{r+2/2}}{1 - \mathcal{P}^{r+2/2}} \]  
(8)

Note that a class \( n \) government will never choose a lower probability since then the probability of repayment is too small for lenders to expect that the government can at the same time be class \( r \) and repay. A larger probability implies a slower speed of learning by lenders and therefore a shorter phase of tranquility.

The expected probability of repayment in the mixing phase of the game can easily be computed using (8) together with the expression for \( p_{T-k-1} \), one finds that is equal to \( \mathcal{P} \). This means that lenders demand a repayment plus interest worth \( \hat{\phi}A - s \) from the government, viz consumption is indeed zero in the mixing phase.

2.2.2 The duration of the mixing phase

The timing of the game can be condensed in the duration of the mixing phase. This is the number of periods required to increase lenders’ prior from \( p_1 \) to \( p_{T-1} \geq \mathcal{P} \). Note that if one defines mixing as \( 0 < \sigma_{T}^n < 1 \), a class \( n \) government will never mix in the last period. The mixing phase might begin already with period 2 and ends with period \( T - 2 \), viz its largest possible length is \( T - 4 \) periods. Its duration is determined by the number of required updates of lenders’ belief. Condition (6) states that the government will gain access to positive loans at all if \( p_1 \) is large enough to allow for \( r - 2/2 \) updates. Denote the duration as \( T \) and note that it is readily computed as:
\[ \left( \frac{\rho}{\phi A - s} \right)^{r+1} \leq p_1 \Leftrightarrow T \leq 2 \frac{\ln(p_1)}{\ln(\rho) - \ln(\hat{\phi}A - s)} - 2 \]  
(9)

For a class \( n \) government, the effective duration of the mixing phase is stochastic due to its mixed strategy. In its decision about its class it uses:

**Lemma 3.** Before the beginning of the mixing phase, a rational government expects that, given it chooses to be class \( n \), it will have to mix for
\[ E_0[T] \equiv T^E = \frac{2}{1 - \mathcal{P}^{r+2}} \left( \frac{\mathcal{P} - \mathcal{P}^{r+2}}{1 - \mathcal{P}} - \frac{T}{2} \mathcal{P}^{r+2} \right) \]  
(10)

**Proof.** See Appendix A.1.
2.3 Government’s Strategy

In this section, there will be statements like "if the government chooses to be class r, the duration of the mixing phase is x". Note that this is no contraction to the implications of lemma 2 that lenders can not infer the class of the government by its observed efficiency. Rather, lenders’ rational strategy may imply $\frac{d p_1}{d \phi} \neq 0$, therefore changing $\phi$ by changing $\alpha_0$ has an effect on the duration of the mixing phase.

Proposition 1. Given the duration of the mixing phase does not depend on the observed efficiency:

- The government will never choose to be class r if $\psi \beta \leq \beta^{T^E - T} (1 - \beta)$.

- If $\psi \beta > \beta^{T^E - T} (1 - \beta)$, the government chooses be class r if $\phi_j$ exceeds a lower bound.
  
  - If $T + 1 - T^E \leq \ln(1 - \beta) - \ln(\phi)/\ln(\beta)$, the upper limit of the interval of possible lower bounds is given by $T = 0$ and the lower limit by $T > 0$.
  
  - If $T + 1 - T^E > \ln(1 - \beta) - \ln(\phi)/\ln(\beta)$, the limits are interchanged.

Proof. By lemma 2 and since $b_{t-1} = 1$, $\alpha_t = \alpha_0$. With $T = 0$, consumption is positive in all production periods (possibly except $T$). Therefore an n class government has no benefit from defaulting before the last period of the game. The benefit of being class r is decreasing with the interest rate charged in period $T$. I use the smallest initial prior compatible with no mixing contracts, viz $p_1 = \mathcal{P}$. This allows to compute the bound related to $T = 0$ by rearranging:

$$
\Delta U_{nm} = \phi_j A - s + \sum_{r=1}^{r=2} \beta^r(\phi_j A - \rho - s) + \beta^r(\phi_j A - \rho \phi_j A/\phi_j A - s) + \beta^{r+1} \psi \frac{\phi_j A}{1 - \beta} \cdots
$$

$$
\cdots - (\phi_j A - (1 - \gamma \alpha_0) s + \sum_{r=1}^{r=2} \beta^{2r}(\phi_j A - \rho - s + \gamma s \alpha_0) + \beta^r \phi_j A) > 0
$$

$$
\iff - \beta^T \left( \frac{\rho}{\phi_j A - s} + s \right) + \beta^{r+1} \psi \frac{\phi_j A}{1 - \beta} > \sum_{r=1}^{r=2} \beta^{2r} \gamma s \alpha_0
$$

$$
\iff - \beta^T \phi_j A + \beta^{r+1} \psi \frac{\phi_j A}{1 - \beta} > 1 - \beta^T \frac{1 - \beta^T}{1 - \beta^2} \gamma s \alpha_0
$$

$$
\iff \phi_j > \frac{1 - \beta^T}{\beta^T A(\beta + 1 - 1)(1 + \beta)} \gamma s \alpha_0 \equiv \hat{\alpha}_0
$$

(11)

If $\psi \beta \leq 1 - \beta$, the efficiency bound is negative and the inequality is reversed, viz the condition can never be fulfilled by a positive $\phi_j$.

Now consider efficiency bound for the case $T > 0$. Lenders might condition $p_1$ on $\hat{\phi}$, therefore the duration of the mixing phase is not necessarily identical for any $\alpha_0 \in [0, 1]$.
This was ruled out in the proposition, but to gain the most general condition possible I already introduce a variable to capture this effect. Let $\Delta$ denote the number of periods the mixing phase is decreased if the government chooses $\alpha_0 = 0$. The efficiency bound can now be computed similar to (11):

$$
\Delta U_m^j = \phi_j A - s + \sum_{\tau=1}^{\tau-\Delta-2} \beta^{2\tau}(\phi_j A - \rho - s) + \beta^{T+1}\psi \frac{\phi_j A}{1-\beta} \ldots \\
\ldots - \left(\phi_j A - (1 - \gamma\alpha_0)s + \sum_{\tau=1}^{\tau-\Delta-2} \beta^{2\tau}(\phi_j A - \rho - s + \gamma s\alpha_0) + \beta^{T-\tau+T^E}\phi_j A\right) > 0
$$

$$
\Leftrightarrow \beta^{T+1}\psi \frac{\phi_j A}{1-\beta} + \sum_{\tau_1=1}^{\tau-\Delta-2} \beta^{2\tau_1}(\phi_j A - \rho - s) > \sum_{\tau_2=0}^{\tau-\Delta-2} \beta^{2\tau_2}\gamma s\alpha_0 + \beta^{T-\tau+T^E}\phi_j A
$$

$$
\Leftrightarrow \beta^{T+1}\psi \frac{\phi_j A}{1-\beta} - \beta^{T-\tau+T^E}\phi_j A + \beta^{T-\tau}\frac{1 - \beta^T}{1 - \beta^2}(\phi_j A - \rho - s) > \frac{1 - \beta^{T-\tau}}{1 - \beta^2} \gamma s\alpha_0
$$

$$
\Leftrightarrow \phi_j A\beta^T \left(\beta\psi \frac{1}{1-\beta} - \beta^{T-\tau+T^E} + \beta^{T-\tau}\frac{1 - \beta^T}{1 - \beta^2}\right) > \frac{1 - \beta^{T-\tau}}{1 - \beta^2} \gamma s\alpha_0 + \beta^{T-\tau}(\rho + s)\frac{1 - \beta^T}{1 - \beta^2}
$$

At this point I restrict to $\Delta = 0$ as done in the proposition. In this case, the last inequality simplifies to

$$
\phi_j A\beta^T \left(\psi\beta - \beta^{T-\tau}(1 - \beta)\right) > \frac{1 - \beta^{T-\tau}}{1 + \beta} \gamma s\alpha_0
$$

**Case 1:** $\psi\beta \leq \beta^{T-\tau}(1 - \beta)$. This means that the discounted utility of true reforms is smaller than the discounted felicity of a default in period $T - \tau + T^E$. In this case the government will never choose to be class $r$.

**Case 2:** $\psi\beta > \beta^{T-\tau}(1 - \beta)$. Here, the discounted felicity of an early default does not exceed the discounted utility of true reforms, therefore the government will consider to do true reforms. In this case one can rearrange to get:

$$
\phi_j > \frac{1 - \beta^{T-\tau}}{\beta^T A(\psi\beta - \beta^{T-\tau}(1 - \beta))(1 + \beta)\gamma s\alpha_0} \equiv \phi(T)\alpha_0
$$

(14)

Obviously, (11) is a special case of (14), namely $\tau = \tau^E = 0$. Therefore, when studying the strategies of lenders and the government, it is sensible to think about the efficiency bound which is given by (14).

The inequality that needs to be verified to show the order of bounds is

$$
\frac{1 - \beta^{T-\tau}}{A\beta^T(1 + \beta)(\psi\beta - \beta^{T-\tau}(1 - \beta))} \leq \frac{1 - \beta^T}{A\beta^T(1 + \beta)(\psi + 1) - 1)}
$$

$$
\Leftrightarrow \frac{1 - \beta^{T-\tau}}{(\psi\beta - \beta^{T-\tau}(1 - \beta))} \leq \frac{1 - \beta^T}{(\psi + 1 - 1)}
$$

14
There is equality if $T^E = T = 0$. A simple approach to see when lhs is smaller is to investigate when the derivative of the lhs with respect to $T$ is negative. One finds:

$$\frac{\partial \text{lhs}}{\partial T} = \frac{\ln(\beta)\beta^{T-T}(\psi\beta - \beta^{T-E-T}(1-\beta)) - \ln(\beta)\beta^{T-E-T}(1-\beta)(1 - \beta^{T-T})}{(\psi\beta - \beta^{T-E-T}(1-\beta))^2}$$  \hspace{1cm} (15)$$

Since $\ln(\beta) < 0$ the derivative is negative if

$$\beta^{T-T}(\psi\beta - \beta^{T-E-T}(1-\beta)) > \beta^{T-E-T}(1-\beta)(1 - \beta^{T-T})$$

$$\Leftrightarrow \psi\beta^{T+1} > \beta^{T^{E}}(1-\beta) \Leftrightarrow \beta^{T^{E}(1-\beta)} > \frac{1-\beta}{\psi}$$

$$\Leftrightarrow T + 1 - T^E < \frac{\ln(1-\beta) - \ln(\psi)}{\ln(\beta)}$$

The case $T + 1 - T^E < \frac{\ln(1-\beta) - \ln(\psi)}{\ln(\beta)}$ is rather intuitive since here the government is more deterred by the prospect of a short phase of tranquility, therefore the no mixing case is relevant for the upper limit.

With this in mind one may be surprised by the possibility that if $T + 1 - T^E > \frac{\ln(1-\beta) - \ln(\psi)}{\ln(\beta)}$, the setting without mixing gives the lower limit while the case with mixing gives the upper one. To make sense of this, note that case 1 applies if $T$ is very large. This is because for $T \to \infty$, (10) implies that $T^E \to 2\psi/1-\psi$, implying that $T^E - T \to -\infty$. Thus, if $T$ is large, $T$ must not be too large to get into case 2. Therefore, the phase of tranquility is large too, giving a low utility weight to events in the mixing phase. In this setting the discounted felicity of a "burst" in consumption due to a default at $T - T + T^E$, i.e. relatively close to the beginning of the mixing phase, exceeds the utility of a continued but moderate consumption stream until $T$. This effect induces the shift of bounds.

The possibility of two orders of bounds is rather discomforting. However, as one might suspect, the case $T + 1 - T^E > \frac{\ln(1-\beta) - \ln(\psi)}{\ln(\beta)}$ is a somewhat remote possibility. To support this suspicion, I shall now discuss plausible numerical values to provide a quantitative dimension to the model.

**Definition 4 (Numerical example).** Let a period be a year and thus $\beta = 0.95$. A government will justify its borrowing with the need to finance $s$. It seems plausible to think on $s$ as being a bit larger than the maximum loan size, which implies that the government has to pay a share of $s$ by other sources of income. I choose $s = 1.2$. Given this view it seems rather implausible that low efficiency governments are able to generate noteworthy consumption if they conduct true reforms, therefore it seems sensible to think on $\nu$ being quite small, I use $\nu = 0.01$.

$\gamma$ describes the share of $s$ that can be used for government consumption when using the mock-up technology. Clearly, a government has to employ some costly cover-up
measures to make sure that lenders are unable to identify the true nature of its spendings. On the other hand, a government that manages to can charge its allies with the task and let the observable work be done by cheap subcontractors, which will allow the government retain a substantial part of the formal spending. Therefore I use the somewhat agnostic value of $\gamma = 0.5$.

Finally, it seems sensible to think on $\psi << 1$ because before $T$, $\phi_j A_t$ includes the necessary output to repay the loan, government consumption is only the residual. I choose $\psi = 0.25$, which makes sure that only low efficiency governments are able to increase their consumption after $T$ relative to the phase of tranquility.

These values imply that the mixing case is the relevant upper bound if $T + 1 - T^E \geq 31$, that is a duration of the reform process of at least 32 years. This time horizon seems very large given the assumption of the model of a continuos reform effort of the government. Therefore, I shall focus on the mixing case as the relevant lower bound in the following.

Now consider the case $\Delta \neq 0$. Denote the efficiency bound with $\Delta \neq 0$ as $\phi(T, \Delta) \alpha_0$.

**Proposition 2.** Given $\phi(T) > 1$, the effect of small $\Delta$ on the efficiency bound is symmetric: The efficiency bound is shifted upward relative to the one computed in proposition 1 if $\Delta < 0$, viz the government is punished for choosing to be class $r$. If $\Delta > 0$, the efficiency bound is shifted downward. The efficiency bound can not arbitrarily be shifted downward by increasing $\Delta$ since there exists an upper bound for the size of the shift.

A sufficiently large "pervasive" punishment, viz a large negative $\Delta$, also shifts the efficiency bound downward. If $\phi(T) < 1$, these properties are reversed.

**Proof.** See Appendix A.2

Before a study of lender’s strategy, it might be helpful to give the model a quantitative feel by using the numerical values discussed above and $T = 20$. First, I compute $A \approx 2.3$ and thus $\frac{2.3}{\lambda} = 0.26$, viz inefficient governments that use the mock-up technology can increase the observed efficiency by about a quarter. Second, I compute $\hat{\phi} \approx 1.3$. Therefore also very inefficient governments with $\phi_j \leq 1.04$ can mimic an efficiency of governments that will choose to be class $r$ with probability one.

### 2.4 Lenders’ Strategy

#### 2.4.1 A basic strategy

Lenders know that their choice of $p_1$, combined with $\hat{\phi}$, determines $T$ due to (9). $T$ and $\hat{\phi}$ in turn determine $T^E$ and those, together with $\Delta$, determine the efficiency bound. These dependencies, combined with the fact that a rational government will anticipate lenders’ strategy, make the identification of an equilibrium strategy for $p_1$ a rather messy
exercise. That said, finding an equilibrium strategy is trivial of a part of the parameter space: If $\hat{\phi} > \overline{\phi} + \gamma / \lambda$, the observed efficiency is so large that it can only be generated by a class $r$ government and offering only no mixing contracts will not change the class choice of such a government. This makes $p_1 = 1$ the only rational choice.

For other observed efficiencies, begin with the case that lenders observe $\hat{\phi} \ll \overline{\phi}$. Trivially, $\phi_j \leq \hat{\phi}$ and thus the government uses $\alpha_0 = (\hat{\phi} - \phi_j) \gamma / \lambda$ to generate $\hat{\phi}$. This means that it will choose to be class $n$ if

$$\phi_j \leq \hat{\phi}(\cdot)(\hat{\phi} - \phi_j) \gamma / \lambda \Leftrightarrow \hat{\phi} \geq \phi_j \left( \frac{\gamma s}{A\phi(\cdot)} + 1 \right)$$

(16)

where $\hat{\phi}(\cdot)$ is the appropriate efficiency bound, yet to be found. Clearly, for $\hat{\phi} \rightarrow 1$, there is some $\hat{\phi}$ for which (16) is not fulfilled. For this government, as well as governments with even smaller $\hat{\phi}$, lenders can be sure that it is class $r$. This means that only governments for which $\hat{\phi} - \phi_j$ exceeds a minimum size choose to be class $n$. If $\hat{\phi} \gtrsim 1$, such governments do not exist, viz: if $\hat{\phi} \gtrsim 1$, there is no doubt that the government is class $r$ and thus lenders will use $p_1 = 1$.

Let us now move to $\hat{\phi}$ that are a bit larger. Suppose that lenders exclude all governments with $\hat{\phi} > \varphi$ from contracts but offer no mixing contracts if they observe $\hat{\phi} \leq \varphi$. This means that for governments with $\phi_j \leq \varphi$, the efficiency bound is $\overline{\phi}\alpha_0$, where $\alpha_0$ has to be chosen such that $\hat{\phi} \leq \varphi$. This necessity tends to scale down the efficiency bound for a subset of governments and makes them choose to be class $r$.

There are two effects when moving $\hat{\phi}$ beyond the region around 1. First, the larger $\hat{\phi}$, the larger the set of governments that could have chosen $\alpha_0 \approx 1$ and will therefore, given $\overline{\phi} \gg 1$, have chosen to be class $n$. Second, $P(\hat{\phi})$ decreases in $\hat{\phi}$. To study the relative strength of these effects rewrite (16) as

$$\phi_j \leq \hat{\phi} \left( \frac{\gamma s}{A\phi(\cdot)} + 1 \right)^{-1}$$

If the efficiency bound is known, this condition allows lenders to compute the probability that the observed government is class $r$ as the probability that the condition is not fulfilled. The probability mass for class $r$ governments is

$$P \equiv P(\phi_j > \hat{\phi} \left( \frac{\gamma s}{A\phi(\cdot)} + 1 \right)^{-1} = \exp \left( -A \hat{\phi} \frac{A\phi(\cdot)}{\gamma s + A\phi(\cdot)} - 1 \right)$$

(17)

I shall denote this probability, that is: the conditional probability that the observed government is class $r$, as $P$. If $P \geq P(\hat{\phi})$ no mixing contracts will be offered and thus $\hat{\phi}(\cdot) = \overline{\phi}$. This is consistent with the assumption of no mixing contracts made above. A simple strategy for lenders is therefore to only offer no mixing contracts, set $\overline{\phi}(\cdot) = \overline{\phi}$ and compute $\varphi$ as the intersection of (17) with $P(\hat{\phi})$.

But what if $\hat{\phi}$ does not fulfill the condition? This is possible when either $\hat{\phi}$ or $\lambda$ are large. Lenders will then prefer to use $p_1 < P(\hat{\phi})$ to decrease the efficiency bound in (17)
and hence increase $\mathbb{P}$. This means that they must decrease $p_1$ with $\hat{\phi}$ to make sure that $\mathbb{P} \geq \mathcal{P}(\hat{\phi})$.

One might be tempted to think that this makes offering contracts for lenders utterly impossible: They have to start with an initial belief smaller than $\mathbb{P}$ to make sure that $\mathbb{P} \geq \mathcal{P}(\hat{\phi})$. To see that this is indeed possible note that the government chooses its class before the first interaction with the lenders. Suppose that lenders can be perfectly sure that the government acts as intended by them. This makes all risk premia charged due to $p_1 < \mathcal{P}(\hat{\phi})$ economic profits and lenders will be tempted to offer more generous contracts after the phase of tranquility to get their hands on some of these profits. The smaller $p_1$ relative to $\mathcal{P}(\hat{\phi})$, the larger the set of lenders that might defect. A rational government expects this, i.e. that the zero consumption phase is shorter than implied by $p_1$ without defection, which increases the efficiency bound. This is the reason why lenders can not be perfectly sure that the government acts as intended by them and helps to understand the role of the sequential game in this setting: It exists due to and removes uncertainty on government’s anticipation of lenders’ strategy. $\mathbb{P}$ is the probability of the government being class $r$ conditional that this uncertainty is nil, which is the case either if no mixing contracts are offered for the whole game or in period $T - 1$. Therefore, $\mathbb{P}$ must always be at least as large as $\mathcal{P}(\hat{\phi})$ while $p_1$ might well be smaller.

To gain profits left out under the simple strategy, lenders could offer contracts with $\mathcal{Y} > 0$ to otherwise excluded governments. However, inspecting (17) reveals that the effect of the efficiency bound on $\mathbb{P}$ is rather small. Therefore, a substantial increase in $\varphi$ requires a substantial $\mathcal{Y}$. Because lenders are in competition, they will still offer no mixing contracts to governments with access to such contracts under the simple strategy, which implies a large $\Delta$ for some governments. By proposition 2 we know that then $\underline{\phi}(\mathcal{Y})$ must be larger than 1, otherwise $\Delta > 0$ counteracts the effect of $\mathcal{Y} > 0$. This limits the range of possible $\mathcal{Y}$ considerably if the reform process is of moderate length and makes the design of a strategy that improves on the simple strategy a rather complex exercise.

Yet, this effort is not necessarily rewarded with large profit opportunities, therefore I shall omit a further exposition of such a strategy. Take again $T = 20$ as example, furthermore suppose $\lambda = 4$, viz a probability that nature draws a government that chooses to reform with probability one of about 30%. To make sure that $\underline{\phi}(\mathcal{Y}) > 1$, lenders must make sure that $\mathcal{Y} \leq 8$. Now suppose lenders are able to set $\mathcal{Y} = 8$ for all governments and to reward a government for choosing to be class $r$ with $\mathcal{Y} = 0$ nonetheless. While this is unrealistically optimistic, it gives an outer limit of the possible improvements lenders can achieve and gives a quantitative feel of the possible gains. The simple and the unrealistically optimistic strategy are depicted in figure 1. The latter would allow to increase the probability mass of governments to which contracts are offered by 6 percent points, which translates to a decrease of the necessary $\mathcal{P}$ from 0.54 to 0.48 or an increase of $\varphi$ from 1.39 to 1.48.
In case a lender uses such an elaborate strategy even though effort and benefit seem to be out of proportion, the use of the simple strategy is a robust choice for other lenders: The elaborate strategy allows to offer contracts to governments excluded under the simple one. That is, the elaborate strategy tries to profit from offering contracts to "risky" governments – lenders who do not prefer to do so, can still offer contracts to "save" governments. They can do this without further ado because the governments they interact with are unaffected by lenders’ choice of strategy and thus, governments’ anticipation of lenders’ strategy choice has no effect on their choice of class.

2.4.2 A supplementing strategy

In the numerical example used for figure 1, \( \bar{\phi} = 1.30 \), implying a "save" observed efficiency of \( \bar{\phi} + \gamma / \lambda = 1.56 \). The intersection \( \varphi \) under the simple strategy is 1.39, therefore a gap between the save observed efficiency and \( \varphi \) exists. With the basic strategy, lenders must exclude such governments. But there is a supplemental strategy that allows to offer contracts to some of these governments.

Thus, assume in this section that \( \varphi < \bar{\phi} \) and denote \( \bar{\phi}^* = \bar{\phi} + \frac{\gamma}{\lambda} \). Suppose that lenders observe a government with \( \hat{\phi} < \bar{\phi} < \bar{\phi}^* \). They know that such an efficiency could have been generated both by a class \( r \) government and by any government with \( \phi_j < \bar{\phi} \) that uses the mock-up technology. Because criterion (17) is not applicable by definition (\( \hat{\phi} \) is too large), lenders can only use their general knowledge about the distribution of \( \phi_j \), that is: they will use \( \exp(-\lambda(\hat{\phi} - 1)) \) as \( p_1 \). If \( \exp(-\lambda(\hat{\phi} - 1)) \geq \mathcal{P}(\hat{\phi}) \), the terminal condition can be reached without mixing and allows lenders to offer contracts. Denote the lower limit for \( \hat{\phi} \) where lenders still offer contracts as \( \bar{\phi}^* | \exp(-\lambda(\hat{\phi}^* - 1)) = \mathcal{P}(\hat{\phi}^*) \).

Offering loans to all governments with \( \hat{\phi} \geq \bar{\phi}^* \), given \( \bar{\phi}^* \geq \bar{\phi} \), is an equilibrium strategy. To see this first note that without the need for lenders to update their belief, \( T = \)
Governments with \( \phi_j \geq \phi^* \) choose to be class \( r \) nonetheless. All governments with \( \phi_j < \phi^* \) choose to be class \( n \) but the probability that nature has drawn such governments is small enough to fulfill the terminal condition. Other lenders can not defect by using a larger intial belief than \( \exp(-\lambda(\hat{\phi} - 1)) \) since this does not reflect the default probability in period \( T \) and thus implies an expected loss for any lender that use this belief. A smaller belief implies \( T > 0 \) and thus too large interest rates at some point of the game.

It is easy to see that \( p_1 \) increases monotonously to 1 for \( \hat{\phi} \) above \( \phi^* \). Consider a government that generates \( \hat{\phi} = \phi^* + f \), where \( 0 \leq f \leq \gamma/\lambda \). Lenders know that this efficiency can only be mimicked by governments with \( \phi_j \geq \phi^* + f - \gamma/\lambda \). The probability mass for class \( n \) governments is readily computed as

\[
P(\text{gov't is class } n) = \int_{\phi^* + f - \gamma/\lambda}^{\phi^*} -\lambda \exp(-\lambda(\hat{\phi} - 1))d\hat{\phi}
\]

\( \Leftrightarrow P(\text{gov't is class } n) = \exp(-\lambda(\phi^* - 1) - (\exp(-\lambda(f - \gamma/\lambda)) - 1) \quad (18)\)

This probability is strictly decreasing in \( f \) and reaches 0 at \( f = \gamma/\lambda \).

Note that governments with \( \phi_j < \varphi \) that can generate \( \hat{\phi} \geq \phi^* \) will do so – because they can use \( \alpha_0 = 1 \) and the relevant efficiency bound is \( \varphi > \phi \). This introduces some complications because all governments in the interval \([\phi^* - \gamma/\lambda, \varphi]\) will choose to be class \( n \), viz the probability to observe a class \( r \) government in this interval is zero given a correct anticipation by governments. Lenders will therefore lower \( \varphi \) until either \( \mathbb{P} = 1 \) under the basic strategy or \( \varphi \leq \phi^* - \gamma/\lambda \). The rationale for the second criterion is obvious, the rationale for the first is as follows: \( \varphi \) can be so small that no mimicking governments with a lower efficiency exist given correct anticipation – because this would require a very small \( \alpha_0 \), which is no attractive option for governments. Because the efficiency bound is much larger and \( \hat{\phi} > \varphi \) can be reached with \( \alpha_0 \approx 1 \), the probability to observe a government in this region is zero. If lenders observe a government nonetheless, this government will have made an error in anticipating the supplemental strategy. Under correct anticipation it would have chosen to be class \( n \), under false anticipation it is therefore likely to be class \( r \), allowing lenders to lend safely to it.

Now suppose that \( \exp(-\lambda(\phi^* - 1)) \equiv \mathbb{P}_s \geq \mathcal{P}(\phi^*) \) requires \( \phi^* < \hat{\phi} \). In this case, lenders can not offer no mixing contracts to all governments with \( \hat{\phi} \geq \phi^* \) because governments with \( \phi_j < \hat{\phi} \) will then choose to be class \( n \). Lenders can again counteract this by choosing \( p_1 < \mathcal{P}(\hat{\phi}) \). But this again puts lenders in a peculiar situation. The smaller the initial belief, the longer the duration of the mixing phase and therewith the smaller the efficiency bound. But the smaller this bound is, the larger \( \mathbb{P}_s \), given governments expect lenders to set \( p_1 \) accordingly. In the last section, I already explained that \( p_1 \) reflects the insecurity whether the government anticipated some lenders to defect, therefore \( p_1 \) must be smaller than \( \mathbb{P} \) and \( \mathbb{P}_s \) in all settings where lenders can defect.

If no mixing contracts imply \( \mathbb{P}_s < \mathcal{P}(\hat{\phi}) \), lenders will therefore decrease \( p_1 \) below \( \mathcal{P}(\hat{\phi}) \) so that the duration of the mixing phase switches to its smallest positive level \( T = 2 \).
Lenders have to make sure that $\Delta = 0$, that is that no government can change the duration of the mixing phase by choosing $\phi_0 > 0$. (Why this is necessary will be explained shortly.) If $\Delta = 0$ is assured, the efficiency bound is $\phi(2,0)$, implying that all governments with efficiencies between $\phi(2,0)$ and $\bar{\phi}$ will also prefer to be class $r$. If now $\mathbb{P}_s \geq \mathcal{P}(\hat{\phi}) | \hat{\phi} \geq \phi(2,0)$, setting $\phi^* = \phi(2,0)$ is an equilibrium strategy, given that again either $\phi \leq \phi^* - r / A$ or $\phi$ is so small that $\mathbb{P} = 1$. The reasons on side of the government have already been discussed. On side of the lenders, no opportunity to defect and profit from it exists. Lenders can neither decrease nor increase $\phi^*$ without accepting expected losses / generating profit opportunities for other lenders.

The critical part of lenders’ strategy is the following: in case of $p_1 \geq \mathcal{P}(\hat{\phi})$ expounded above, $p_1$ can readily be increased with $\hat{\phi}$ since governments’ choice of class is unchanged by a larger $p_1$. With $p_1 < \mathcal{P}(\hat{\phi})$, $p_1$ can be increased too fast with $\hat{\phi}$ in the sense that $\mathcal{T} = 0$ is reached even though $\hat{\phi} < \phi(2,0) + r / A$. If this is anticipated by governments, the correct efficiency bound of governments with $\phi_i \leq \bar{\phi}$ is given by $\phi(0, -2)$. This is because if they choose to be class $r$, they generate an observed efficiency that forces them into a production period with zero consumption while this can be avoided if they choose to be class $n$. If $\phi(2,0) > 1$, the correct bound is by proposition 2 larger than $\bar{\phi}$, therefore these government choose to be class $n$, making again $\mathbb{P}_s < \mathcal{P}(\hat{\phi})$. For this reason, lenders must make sure that $p_1$ is increased in a way that $\mathcal{T} = 0$ is not reached until $\hat{\phi} > \phi(2,0) + r / A$. This makes sure that governments with $\phi(2,0) \leq \phi_j \leq \bar{\phi}$ face $\Delta = 0$ and this in turn makes the efficiency bound robust to governments’ behavior.

If the increase to $\mathcal{T} = 2$ by decreasing $p_1$ to $p_1 \leq \mathcal{P}(\hat{\phi})$ is not sufficient to reach $\mathbb{P}_s \geq \mathcal{P}(\hat{\phi})$, lenders will reduce it further to $p_1 \leq \mathcal{P}(\hat{\phi})^{\gamma / \beta}$ and set $\phi^* = \phi(4,0)$. If now $\mathbb{P}_s \geq \mathcal{P}(\hat{\phi})$ is reached, this is again an equilibrium strategy if lenders make sure that $\Delta = 0$ for the same reasons as for $\mathcal{T} = 2$. This means that the initial belief must be increased in $\hat{\phi}$ such that as long as $\hat{\phi} \leq \phi(4,0) + r / A$, $\mathcal{T} = 4$ and for $\hat{\phi} \leq \phi(2,0) + r / A$, $\mathcal{T} = 2$. It should be obvious that lenders will decrease $p_1$ accordingly until for some $p_1 \leq \mathcal{P}(\hat{\phi})^{\gamma / \beta}$, $\mathbb{P}_s \geq \mathcal{P}(\hat{\phi})$. This is possible if and only if for some $\hat{\phi}$

$$\exp(-\lambda(\hat{\phi} - 1)) \geq \mathcal{P}(\hat{\phi}) = \frac{\rho}{\hat{\phi}A - s}$$

to solve for $\lambda$, I approximate this inequality as:

$$\frac{1}{1 + \lambda(\hat{\phi} - 1)} \geq \frac{\rho}{\hat{\phi}A - s} \iff \lambda \leq \beta A + \frac{\gamma s \beta}{\hat{\phi} - 1}$$

For the numerical example, $p_1$ and $\mathcal{P}(\hat{\phi})$ intersect at 1.06. The next efficiency bound below is the one for $\mathcal{T} = 10$. This makes $\phi(10,0) = \phi^*$ the appropriate lower bound, which is 0.88. This allows to cover the complete range of government, lenders will therefore only use the supplemental strategy.
2.5 Conclusion on the model

I demonstrated that lenders will offer contracts to governments even though they know that there is a positive probability that the government they interact with does not truly reform and might therefore default in some future period. Lenders’ strategy does make sure that at least a large subset of governments has access to loan contracts. Furthermore, lenders’ strategy makes sure that a large subset of these governments will choose to be class r, given governments do not expect lenders to defect. However, such a defection of lenders can not and must not be ruled out to allow for $T > 0$, viz zero consumption periods that are lenders’ instrument to discipline governments. This implies that market discipline in favor of reforms is necessarily imperfect.

The possibility of two interest rate regimes, viz a phase of tranquility and a zero consumption phase, indicates that sudden jumps in interest rates are not necessarily a sign of drastic corrections in lenders’ beliefs about a government. Yet, such jumps in the context of the EMU debt crisis have prompted some authors to conclude that market discipline simply did not work.\(^3\) While there may exist numerous empirical pattern that support this claim, testing it requires an alternative hypothesis. In the next section, I shall present such a hypothesis.

3 Application: Bail Outs in the EMU

In this section, I augment the model with a second country: beside the country from the basic model, denoted as country $C$, there is a second country $E$ that has already completed its reforms and has a very large output.

**Assumption 12** (Preferences). $E$ has the same preferences as $C$.

**Assumption 13** (Resources). $E$’s income is given by

$$y^E_t = y^E + z^C_t M$$

for $t = 0, \ldots, \infty$

where $z^C_t$ describes $C$’s decision to repay similar to the basic model, $y^E >> \phi A$ and $M$ a positive constant.

A simple intuition for why $z^C_t M$ is entering $E$’s income is the following: If $C$ defaults, its output collapses. Firms of $E$ are active in $C$, therefore a default will decrease $E$’s tax receipts. Furthermore, $z^C_t = 0$ means that $C$’s reforms failed, implying long term negative consequences for the economy and thus perpetually smaller business opportunities for $E$’s firms. If both countries are in some form of political union $E$ might also be forced to provide transfer payments from $E$ to $C$.

**Assumption 14.** $E$ has the same abilities as lenders: it can observe the same variables, knows the same parameters and can also offer contracts to $C$.

\(^3\)See, e.g. Tichy (2012)
This assumption implies that \( E \) has very limited capabilities to make sure that \( z^C_t = 1 \) for all \( t \). A simple approach to make this sure is the following: \( E \) could replace lenders at the beginning of the mixing phase and offer more generous contracts to \( C \) that allow for the same consumption during the mixing phase as in the phase of tranquility. If this is the case, \( C \) will never default before period \( T \). But this can have an adverse effect: if \( E \) decreases \( T \) considerably below what is implied by lender’s strategy expounded in the basic model, \( C \) will choose to be class \( n \) with probability 1 if it expected \( E \) to replace lenders when choosing its class. However, this adverse effect affects \( E \) in the distant future, therefore \( E \) might be willing to tolerate it. To gain an explicit condition, investigate the utility difference between replacing lenders and accepting a default in \( T \) and remaining passive and accepting whatever the outcome of the interaction between \( C \) and its lenders is:

\[
\Delta U = \left\{ \beta^{T-T-1}(-1+M) + \beta^{T-T} \left( \rho + M \right) + \beta^{T-T+1}(-1+M) + \cdots + \beta^{T-1}(-1+M) \right\} \cdot \frac{\mathbb{P}\left( \sum_{\tau_1=T-T-1}^{\infty} \beta^{\tau_1} M \right)}{E \text{ replaces lenders}} + \left( 1 - \mathbb{P}\right) \left( \beta^{T-T-1} M + \beta^{T-T} M + \cdots + \beta^{T-T+T_E-1} M \right) \cdot \frac{\mathbb{P}}{E \text{ remains passive}}
\]

\[
\Leftrightarrow \Delta U = \frac{\beta^{T-T-1} - \beta^{T-1}}{1 - \beta^2} - \beta^{T-1} - \beta^{T-T} \rho - \frac{\beta^{T-T+T_E} - \beta^T}{1 - \beta} M - \mathbb{P} \beta^{T-T+T_E} \frac{1}{1 - \beta} M
\]

\[
\Leftrightarrow \Delta U = \left( 1 - \mathbb{P}\right) \frac{\rho^{T-T_E} - 1}{1 - \beta} M - \mathbb{P} \frac{1}{1 - \beta} M - \rho > 0 \Leftrightarrow (1 - \mathbb{P}) \rho^{T-T_E} > 1 + \frac{\rho - 1}{M}
\]

where \( \mathbb{P} \) is the conditional probability that \( C \) is class \( r \) from the basic model. Obviously, this condition is easy to fulfill if \( \mathbb{P} \) is relatively small (viz, if \( \phi \) of \( C \) is large), \( M \) is large or \( T - T_E \) is large.

In the real world, governments indeed replaced capital market finance for a number of other governments during the EMU debt crisis. But previously, a bail-out occurred, making this replacement quite different to the strategy sketched above. During the first ten years of the EMU an important property of the EMU bond market was the near absence of interest rate spreads compared to recent levels, viz a compression of risk premia.\(^4\) In the model, a country that pays no risk premium is not in the mixing phase and will therefore not default. Thus, if one reads the beginning of the EMU in the terms of the model, the compression of risk premia suggests that the transition has introduced some mechanism that produces the pattern \( E \) prefers without replacing the lenders. This mechanism is the possibility of a bail-out.

\(^4\)For an overview of the EMU bond market at the transition to the EMU see Pagano and Von Thadden (2004).
A number of researchers found little to no evidence that, during the first ten years, the remaining EMU spreads reflected macroeconomic conditions or the risk of a sovereign default.\(^5\) This can be explained with the argument that investors knew that large and wealthy EMU countries will bail out smaller ones if they are in trouble, which makes their idiosyncratic risks unimportant. The problem with this argument is that it is in stark contradiction to EU law. Article 125 of the Treaty on the functioning of the European Union states: "(…) A Member State shall not be liable for or assume the commitments of central governments (…) of another Member State (…)".

Therefore one may wonder why there seemed to be a strong consensus amongst bound traders and investors that this particular clause is not credible. An argument for this that seems to be mirrored by the factual events of the EMU debt crisis has been made by Bovenberg et al. (1991, p 381): "By eliminating currency risk, a monetary union would encourage EMU residents to invest in debt instruments issued by governments of other member countries. The increased exposure of their residents would increase the pressures on EMU governments to bail out a member in financial distress.".\(^6\) Additionally to a number of ex ante plausible counter arguments\(^7\) one may wonder about the degree of confidence that investors must have had into such a mechanism. One might be tempted to think that their confidence must have been rather perfect because a substantial expected probability that the clause might be true should imply a considerable risk premium.

The model helps to see that this is not necessarily the case. In the model, is indeed possible to make the risk premium drop to zero for a number of periods even though lenders know that a bail-out will not come for sure. I shall model the bail-out as follows:

**Assumption 15.** The government in C decides on \(z_C^t\) but calls E before publishing that it will set \(z_C^t = 0\). E repays C’s loan plus interest with probability \(\sigma_E > 0\). \(\sigma_E\) is publicly known and it is publicly known whether a bail-out occurred.

Denoting lenders’ belief used for the last loan as \(p_C^{T-1}\) this changes (2) to:

\[
\begin{align*}
\frac{p_C^{T-1}}{C \text{ repays}} + (1 - p_C^{T-1})\sigma_E & \geq \frac{\rho}{\phi A - b_{T-1}} \quad \Leftrightarrow \quad p_C^{T-1} \geq \frac{\rho}{(1 - \sigma_E)(\phi A - b_{T-1})} - \frac{\sigma_E}{1 - \sigma_E} \equiv \mathcal{P}_C(\phi)
\end{align*}
\]

Since \(\sigma_E\) is constant, \(b_{T-1} = b_{T-3} = \cdots = 1\) for the same reasons as in the basic model. C is still deciding whether to repay the loan and thus lenders can learn about C’s class by the same method as in the basic model. Denoting the mixing probability chosen by

\(^5\)For evidence on the missing relevance of the default risk in spreads see Balli (2009) and Geyer et al. (2004). Gómez-Puig (2008) finds evidence that hinds into the same direction, although less conclusive.

\(^6\)A closely related argument was also made by Buiter et al. (1993).

\(^7\)E.g. that EMU governments are deprived from the possibility of easing the burden of debt by high inflation, which should c.p. decrease their willingness to borrow.
$C$ in case $C$ is class $n$ as $\sigma_r^C$, the feasibility condition for repayment in periods before $T$ can be stated as:

$$\frac{p_{T-j}^C + (1 - p_{T-j-1}^C)\sigma_{T-j}^C}{p(C \text{ repays})} + \frac{(1 - p_{T-j-1}^C)(1 - \sigma_{T-j}^C)\sigma^E}{P(E \text{ pays for } C)} \geq \frac{\rho}{\phi A - s}$$

$$\Leftrightarrow p_{T-j-1}^C \geq \frac{\rho}{(1 - \sigma_{T-j}^C)(1 - \sigma^E)(\phi A - s)} - \frac{\sigma_{T-j}^C + \sigma^E - \sigma_{T-j}^C\sigma^E}{(1 - \sigma_{T-j}^C)(1 - \sigma^E)}$$

Which can also be written as

$$p_{T-j-1}^C \geq \frac{1}{1 - \sigma_{T-j}^C}(\mathcal{P}^C - \sigma_{T-j}^C) \Leftrightarrow p_{T-j-1}^C(1 - \sigma_{T-j}^C) + \sigma_{T-j}^C \geq \mathcal{P}^C \quad (21)$$

Lenders still update their belief using Bayes’ rule:

$$p_{T-j+1}^C = \frac{p_{T-j}^C}{p_{T-j-1}^C(1 - \sigma_{T-j}^C) + \sigma_{T-j}^C}$$

Combining this with (21) yields the familiar condition

$$\frac{p_{T-j-1}^C}{p_{T-j+1}^C} \geq \mathcal{P}^C$$

Therefore, the mechanics and rules according to which lenders’ belief is updates are similar to the basic model. Clearly, this implies that $C$ will set its mixing probability by the same rule as in the basic model.

Thus, allowing for the possibility of a bail-out only decreases the necessary probability that a government is class $r$ in period $T$. This means in particular that lender’s strategy is unchanged, they merely have to replace (2) with (20). With the possibility of a bail-out, lenders will be satisfied with a smaller $\mathcal{P}$ or $\mathcal{P}_s$, which implies that $\varphi$ and $\phi^*$ will be larger. A larger $\phi^*$ implies that the duration of the mixing phase with $\sigma^E > 0$, denoted as $\mathcal{T}^C$, is smaller than that of the basic model (still denoted as $\mathcal{T}$).

As one might suspect, using $\sigma^E > 0$ instead of a replacement of lenders might be a rather cheap way to push the expected period of a default into the future. To investigate this suspicion formally, repeat the exercise that yielded (19). I shall denote the expected duration of a mixing phase as $E[\mathcal{T}]$, the conditional probability that $C$ is class $r$ under a bail out as $\mathbb{P}_{\sigma^E > 0}$ and without a bail out as $\mathbb{P}_{\sigma^E = 0}$. The utility difference is now:
skepticism that these reforms have economic substance. With the introduction of the model, the interest rate spreads in these days can be understood as a measure of lenders’ and early 2000s years (see criteria. And indeed, academic research reports evidence that most EMU countries un-
reform, both to avoid a collapse in growth and employment and to meet the Maastricht 
countries faced numerous economic problems during the 1990s that required them to
mixing phase for core countries (were already in the zero consumption phase. With the beginning of the EMU, wealthy
for if one allows for a change in some C.

The model can generate the rough pattern of interest rates at the beginning of the EMU
if one allows for a change in \( \sigma^E \) and assumes that lenders use the supplemental strategy
for \( C \). Before the start of the EMU, peripheral countries (\( C \)) generated \( \hat{\phi} \geq \phi^* \) and
were already in the zero consumption phase. With the beginning of the EMU, wealthy
core countries (\( E \)) found \( \sigma^E > 0 \) to be beneficial. This decreased the duration of the
mixing phase for \( C \) and made risk premia disappear for a while. The financial crisis
was the moment of truth in which previous reform efforts had to show their worth, with
lenders demanding a large risk premium due to their doubts about the quality of these
reforms. This put \( C \) back in a zero consumption regime, followed by a de facto default
with bail-out by \( E \) shortly after.

To put a bit more meat on the bones of this story, recall that many peripheral EMU
countries faced numerous economic problems during the 1990s that required them to
reform, both to avoid a collapse in growth and employment and to meet the Maastricht
criteria. And indeed, academic research reports evidence that most EMU countries un-
dertook more structural reforms than the average OECD country during the late 1990s
and early 2000s years (see Alesina et al., 2008, Elmeskov et al., 2006). In terms of the
model, the interest rate spreads in these days can be understood as a measure of lenders’
skepticism that these reforms have economic substance. With the introduction of the

\[
\Delta U = \left\{ \mathbb{P}_{\sigma^E \neq 0} \left( \sum_{t_1=0}^{\infty} \beta^{t_1} M + (1 - \mathbb{P}_{\sigma^E \neq 0}) \left( \sum_{t_1=0}^{T_T} \beta^{t_1} M + \sum_{t_1=T_T+1}^{T_T+E(T_C)-1} \beta^{t_1} M - \beta^{T_T+E(T_C)} R_{T_T-1} \sigma^E \right) \right) \right\}
\]

\[
= \left\{ \mathbb{P}_{\sigma^E \neq 0} \left( \sum_{t_1=0}^{\infty} \beta^{t_1} M + (1 - \mathbb{P}_{\sigma^E \neq 0}) \left( \sum_{t_1=0}^{T_T} \beta^{t_1} M + \sum_{t_1=T_T+1}^{T_T+E(T_C)-1} \beta^{t_1} M \right) \right) \right\}
\]

\[
\Leftrightarrow \Delta U = \left\{ \mathbb{P}_{\sigma^E \neq 0} - \mathbb{P}_{\sigma^E = 0} \right\} \left\{ \frac{1}{1 - \beta} \left( \frac{1 - \beta^{T_T+1}}{1 - \beta} \right) M - (1 - \mathbb{P}_{\sigma^E \neq 0}) \beta^{T_T+E(T_C)} R_{T_T-1} \sigma^E \ldots \right\}
\]

\[
\ldots + (1 - \mathbb{P}_{\sigma^E \neq 0}) \frac{\beta^{T_T}}{1 - \beta} \left( \beta - \beta^{T_T+E(T_C)} \right) M - (1 - \mathbb{P}_{\sigma^E \neq 0}) \frac{\beta^{T_T}}{1 - \beta} \left( \beta - \beta^{E(T_C)} \right) M \right\}
\]

Two effects are at work: first, \( \sigma^E > 0 \) tends to shift down the duration of the mixing
phase, therefore \( T_C \leq T \). Second, a shorter mixing phase also means that a class \( n \)
government tends to default after a shorter number of zero consumption periods. For
this reason, it is not clear whether \( T - T_C + E(T_C) > E(T) \) applies, i.e. if the utility
difference it indeed positive. It is however easy to gain a condition when this is the case:

\[
\Delta \mathbb{P} + (1 - \mathbb{P}_{\sigma^E = 0}) \frac{1 - \beta}{M} \beta^{T-T_C+E(T_C)} R_{T-1} \sigma^E + \beta^{T-T} (1 - \mathbb{P}_{\sigma^E = 0}) \Phi(\sigma^E = 0) \ldots
\]

\[
< \Delta \mathbb{P} (1 - \beta^{T-T+1}) + \beta^{T-T} (1 - \mathbb{P}_{\sigma^E > 0}) \Phi(\sigma^E > 0)
\]

\[
\Leftrightarrow \Delta \mathbb{P} \beta + (1 - \mathbb{P}_{\sigma^E > 0}) \sigma^E (1 - \beta) R_{T-1} \frac{\beta^{T-E(T)-T_C-E(T_C)}}{M} \ldots
\]

\[
< (1 - \mathbb{P}_{\sigma^E > 0}) \Phi(\sigma^E > 0) - (1 - \mathbb{P}_{\sigma^E = 0}) \Phi(\sigma^E = 0)
\]

(22)
EMU, the interest rate spreads were compressed to near zero while doubts on peripheral countries’ reforms prevailed. For example, Spain was listed as top ten reformer for 2003 due to its reform measures in 2001 / 2002 (World Bank, 2005, table 1.1) with an ease of doing business rank of 30 (out of 180 countries). In the subsequent years, this praise got silent, Spain’s rank dropped to 38 and remained there. Shortly after the beginning of the financial crisis, peripheral EMU countries found themselves in a very similar situation to the mid 1990s, viz under massive pressure to tackle the newly highlighted backlog of reforms and paying high risk premia. In case of Spain, its ease of doing business rank dropped to 49 with beginning of the financial crisis in 2008/2009, coinciding with a tripling of the interest rate spread to Germany.

One question remains: What is the role of the no bail out clause? Suppose $E$ could fool $C$ and in the sense that it expects $\sigma^E = 0$ when choosing its class. This would leave $\mathbb{P}$ on the level of the basic model and (22) would simplify to

$$
\sigma^E \frac{(1 - \beta)R_{T-1}}{\beta^{(T - E[T] - (T_C - E[T_C])}} < \frac{\Phi(\sigma^E > 0)}{\Phi(\sigma^E = 0)}
$$

which is very easy to fulfill. This is because introducing $\sigma^E > 0$ later on would be close to a free lunch for $E$: It does not loose probability mass for $C$ choosing to be class $r$, yet it can increase the duration of the phase of tranquility, viz reduce the utility loss in case $C$ is class $n$. However, rational lenders and a rational government in $C$ know that $E$ would benefit from this dynamic inconsistency and will therefore not believe in any pledge of $E$ on $\sigma^E = 0$, even if it is written in law.

### 4 Conclusion

I developed a stylized model in which governments need loans to finance reforms. A government may misuse these funds by doing bogus-reforms and consuming more instead, which implies a default of the government at some point in time. Therefore lenders have to discipline governments in favor of true reforms. This works, but far from perfect. I also demonstrated that by enhancing the model with a second country that may bail out the first, the model is capable of generating an U-shaped pattern of interest rates as observed in the first decade of the EMU. Furthermore, the model provides an economic rationale on why the EU’s no bail out clause was never credible for lenders and peripheral EMU countries.

Due to the highly stylized nature of the model, the mechanisms it highlights are only one of possibly many ingredients necessary for the understanding either the sovereign defaults mentioned in the introduction or the pattern of intra EMU interest rate spreads. It leaves some important issues for future research: Most importantly, the model has be translated into a stochastic environment. Closely related to this is the abolishment of
the assumption that even one period where government spends less than $s$ on reforms ultimately leads to a sovereign default.

A core message of the model is that there are two sources of risk for lenders: First, a risk due to insecurity about government’s anticipation of lenders’ lending behavior. This insecurity forces lenders to charge risk premia and thus disciplines the government. Second, a risk due to government’s choice of class given the first risk is nil. This risk determines whether lenders will offer contracts at all and can be influenced by lenders via their approach on the first type of risk, viz when and how risk premia are charged. In the real world, more types of risk may exist. Therefore, some authors’ conclusion that jumps in interest rates are a clear sign of financial markets’ missing ability to evaluate risks correctly seems heroic. Because not all risks are necessarily always part of the risk premium (in the model, the first risk is not during the phase of tranquility, the second risk is relevant only for the last loan) sudden jumps or collapses of a risk premium may have a rationale. Before concluding that financial market’s behavior is irrational, one has to rule out that those jumps were due to changes in the composition of risk premia.

References


A Appendix

A.1 Proof of Lemma 3

The government knows that

<table>
<thead>
<tr>
<th>number of mixing periods</th>
<th>probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1 - $\sigma^n_{T-\tau}$</td>
</tr>
<tr>
<td>2</td>
<td>$\sigma^n_{T-\tau}(1 - \sigma^n_{T-\tau+2})$</td>
</tr>
<tr>
<td>4</td>
<td>$\sigma^n_{T-\tau}\sigma^n_{T-\tau+2}(1 - \sigma^n_{T-\tau+4})$</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>$T - 2$</td>
<td>$\prod_{t=\frac{T-2}{2}}^{\frac{T-2}{2}} \sigma^n_{2t}(1 - \sigma^n_{T-4})$</td>
</tr>
<tr>
<td>$T$</td>
<td>$\prod_{t=\frac{T-2}{2}}^{\frac{T-2}{2}} \sigma^n_{2t}(1 - \sigma^n_{T-2})$</td>
</tr>
</tbody>
</table>

This allows the express the expected number of mixing periods as

$$
E[T] = 2\sigma^n_{T-\tau}(1 - \sigma^n_{T-\tau+2}) + 4\sigma^n_{T-\tau}\sigma^n_{T-\tau+2}(1 - \sigma^n_{T-\tau+4}) + \cdots + T \prod_{t=\frac{T-2}{2}}^{\frac{T-2}{2}} \sigma^n_{2t}(1 - \sigma^n_{T-4})
$$

$$
= 2\sigma^n_{T-\tau} + 2\sigma^n_{T-\tau}\sigma^n_{T-\tau+2} + 2\sigma^n_{T-\tau}\sigma^n_{T-\tau+2}\sigma^n_{T-\tau+4} + \cdots + 2 \prod_{t=\frac{T-2}{2}}^{\frac{T-2}{2}} \sigma^n_{2t} - T \prod_{t=\frac{T-2}{2}}^{\frac{T-2}{2}} \sigma^n_{2t}
$$

Now note that (8) can also be written as $\sigma^n_{T-k} = \mathcal{P}\frac{1 - \mathcal{P}^{T-k}}{1 - \mathcal{P}^{T-k+1}}$ which allows to simplify $E[T]$ to

$$
E[T] = 2\mathcal{P}\frac{1 - \mathcal{P}^{T}}{1 - \mathcal{P}^{T+1}} + 2\mathcal{P}^2\frac{1 - \mathcal{P}^{T-1}}{1 - \mathcal{P}^{T+1}} + \cdots + 2\mathcal{P}^{T-k}\frac{1 - \mathcal{P}^{T}}{1 - \mathcal{P}^{T+1}} - T \mathcal{P}^T \frac{1 - \mathcal{P}}{1 - \mathcal{P}^{T+1}}
$$

$$
= \frac{2}{1 - \mathcal{P}^{T+1}} \left( \mathcal{P}(1 - \mathcal{P}^{T/2}) + \mathcal{P}^2(1 - \mathcal{P}^{T/2-1}) + \cdots + \mathcal{P}^{T/2}(1 - \mathcal{P}^2) - \frac{T}{2} \mathcal{P}^{T/2}(1 - \mathcal{P}) \right)
$$

$$
= \frac{2}{1 - \mathcal{P}^{T+1}} \left( \mathcal{P} - \mathcal{P}^{T/2} - \frac{T}{2} \mathcal{P}^{T/2} \right)
$$
A.2 Proof of Proposition 2

Suppose a government gets access to contracts with $T = T_n$ if it is class $n$ and that it can reduce $T_n$ by $\Delta = w \leq T_n$ if it chooses to be class $r$. Assume $\nu = 0$ and rewrite (13):

$$\phi_j A\beta^T \left( \frac{1}{1-\beta} - \beta^{-T+T^e} \right) + \phi_j A\beta^{T-T} \frac{1-\beta^w}{1-\beta^2} > \frac{1-\beta^{T-T}}{1-\beta^2} \gamma \alpha_0 + A\beta^{T-T} \frac{1-\beta^w}{1-\beta^2}$$

It is easily verified that effect of $w$ on $e$ is approximately linear with positive slope around $w = 0$. This means if $w \lesssim 0$, $e \lesssim 0$ while for $w \gtrsim 0$ $e \gtrsim 0$ too. Yet, $\lim_{w \to -\infty} e = -\infty$ while $\lim_{w \to \infty} e$ is a finite positive number.

The bound computed in proposition 1 is $\phi(T)\alpha_0 = b/a$. The bound with $w \neq 0$ is $\phi(T_n, w)\alpha_0 = b+e/a+e$. The difference between these bounds is $b/a - b+e/a+e = e[\phi(T)-1]/a+e$. This implies that there is a discontinuity around $-e = a$. In the following, suppose $\phi(T) > 1$. If $e < 0$ (the government is punished for being class $r$ by an increase of $T$) and $a + e > 0$ the difference is negative, viz the efficiency bound is shifted upwards. If $e > 0$ (the government is rewarded) the difference is positive, viz the efficiency bound shifts downwards. For $w \to \infty$, $e \to A\beta^{T-T} \frac{1}{1-\beta^2}$ which implies a difference of $\frac{\phi(T)-1}{b+e/a+e} > 0$.

Finally, if $e < 0$ and $a + e < 0$, the difference is positive again and might be very large if $a + e \approx 0$. For $w \to \infty$, the difference converges to $\frac{\phi(T)-1}{a} > 0$.

Allowing for $\nu > 0$ implies that the $e$ in the denominator of $b+e/a+e$ is smaller than the $e$ in the numerator. Therefore the difference and therewith the shifts are a bit larger than under the approximation but the general pattern is unchanged. If $\phi(T) < 1$, the sign of $e[\phi(T)-1]/a+e$ is flipped relative to the results above.